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Experimental Investigation and Method of Mathematical Modeling of Electrostatic Discharges

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The danger of discharges on dielectric materials in a space environment is usually estimated by testing samples in vacuum installations. It has been deduced from experiments that a correspondence of order is between the topology of the discharges and the group of pulse invariants in the grounded circuit of the sample substrate. These mathematical properties determine a model of discharge by the symplectic product of a logical tensor of an embedded topomorphic discharge structure by the algebraic tensor of a pulse group from the parametric series. The general group of numbers with a transfinite extension of tensor indexes in the discharge model connects three classes of mathematical objects: topology of connections, algebra of properties, and numbers of their common order. This extension of property variety in general mathematics was forecast by Poincaré. The model of discharge is defined on a hyperplane embedded into a space of complex variables with an extension of polysheet Riemann's surface over the group of sliding that is determined by the Clifford virtual quantifier $\delta = \sqrt{0}$ with the argument of the circumferential group. The analytical function of the model on the hyperplane extends over basis parameters of symplectic generalized functions into a series of wavelet pulses and harmonics. Therefore, the complicated pulse is identified by parametric groups of the previous expansions. This is a clef to the identification of discharges and creating the generalized database of tests for materials.

Nomenclature

C = electric capacity, F
E_e = energy of electron, keV
I = standard pulse of unit amplitude, A
i_e = electron flux, A/sm^2
$J(t)$ = pulse function in load circuit, A
q = local density of charge, C/m^2
R = electric resistance, ohm
S_τ = vector of strobe shift times in the discharge, s
t = process duration, s
U = voltage, V
V = potential relative to space, V
w = energy of discharge pulse, J
Y = signal of complicated discharge, A
ρ = thickness of dielectric film, μm
τ = local time, s

Introduction

ELECTROSTATIC discharges (ESDs) on spacecraft (SC) create current-voltage pulses and radio-frequency interference dangerous for electronic equipment and, with time, they can even lead to degradation of material properties [1]. The tendency of materials to spark in space is checked in experimental installations when irradiating samples by accelerated electron beams [2]. To estimate the capability of materials to withstand the effects of charging, a model of standard discharges invariant to conditions of different installations is necessary. Standard pulses are also used in tolerance tests of electronics to ESD. The level of ESD danger can be estimated on the basis of physical models for their development in the operating conditions of SC [3] and the methods of mathematical modeling of ESD pulses with accounts of topology, dielectric properties of surfaces, and load impedance in the structure of elements. Multiple classification of high-energy electron interference with samples has revealed group properties of ESD and stable connection of pulse structure in the grounded circuit with the topology of ESD groups on the sample surface [4]. Those properties of specific samples determine a group of invariants for discharge pulses, and the properties of the local ESD pulse represent those of the group. The properties of the local ESD basic model are dependent on the braking distance of electrons in the dielectric, the field diffusion drift rate, permittivity, and the activation energy for breakdown. As a whole, the properties of discharge are reflected in its pulse and topology structure with groups. The recursion of the discharge function group can be described by a group of similar electric and topologic equivalents for invariants of the standard pulse. The topology of ESD agrees with the structure of pulses in the wavelet expansion with the parametric group, and its invariants in similar discharges correspond to the standard pulse in a local cell. The automorphism of discharge chain topology forms a multiplicative semigroup for the embedding of tensor functions with the equivalent discharge cell properties. Therefore, the multiple pulses can be described as a simplex of the parametric extension for the group of model pulses obtained in experiments for a specific sample. The coordinated elements of the groups of basic Fourier and wavelet images form a logical cortege of the universal module for the parametric extension of multiple discharge signals. The parameters of the module extension coordinate the load impedance of the pulse with the topology of the discharge group development. The charging processes in the local cells form a scalar electrostatic potential field on the sample, and the maximums of its gradient determine the possible places of the discharges. Such mathematical description requires combined logical–algebraic operations. In a virtual complex space (VCS) of generalized functions, the model of discharge can be written in the form $Y = T \cdot J$, which coordinates the topology T with the metatensor J; that is, it determines the essence of the discharge. The identification of multiple discharges by stable standards allows the retrieval of experimentally founded heuristics of pulse codification

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by the level of danger for the SC. In the logistic models, the forecast of the discharge development topology is ensured by an external function of the oracle, which requires a foundation for its heuristics. In the complex model of discharge, the topology frame binds relations in a unitary logical–algebraic function of the field gradient tensor and the conditions for breakdown.

Properties of Pulsed Discharges

The process of discharge in Fig. 1 consists of gradual charge accumulation in a medium (equivalent capacity C_p) and its subsequent fast avalanche throwoff through the breakdown gap to the substrate grounded through the resistor R_g and capacitor C_g, and it has cycloramalike pulses in the relaxation oscillator, in Fig. 2.

The electrostatic potential of the sample surface is measured by a field sensor, and the discharges are registered by current pulses in the load circuit as well as by light flashes from discharges on the plate.

The simple breakdowns arise from charged conducted surfaces (for example, from metals) and have, according to an equivalent capacitor, stable characteristics obtained by computation or from experiments (Fig. 3). Such discharges can appear in the various equally loaded places with the general tensor equivalent.

The pulse in Fig. 3 can be approximated by the function $U(t) = P(t) \cdot \exp[f(t)]$. Its expansion in operand parameters (the rate of pulse rise $k_1 = \Delta U/\Delta t_1$ and that of the logarithmic fall $k_2 = \Delta U/\Delta t_2$) determines a pulse model of near-triangular form that serves as a standard for the basic expansion of a multiple discharge model in generalized parameters. The heterogeneous properties of dielectrics, as well as electron flux fluctuations, lead to uneven charging of the surface. Therefore, the potential evens out slowly after a local discharge, and the next discharge can only arise elsewhere. The breakdown from dielectric to vacuum creates a plasma outburst and induces a positive pulse on the substrate, whereas the discharge to the substrate creates a negative pulse (Fig. 4).

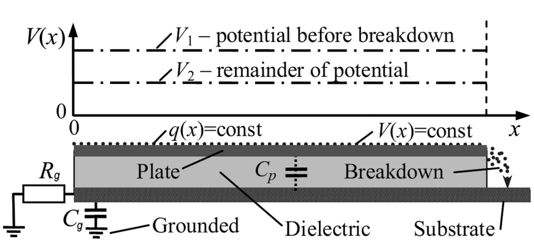


Fig. 1 **Discharge from a metal sample.**

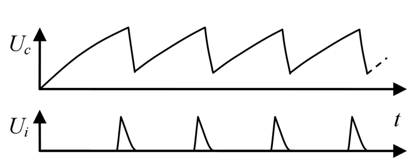


Fig. 2 **Process of charging U_c and discharge pulses U_i.**

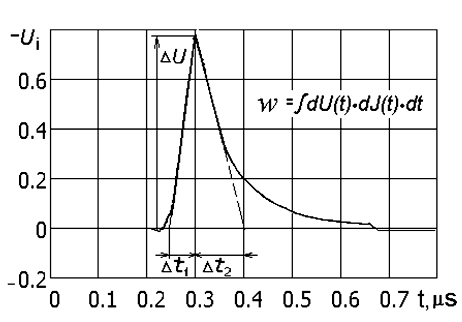


Fig. 3 **A pulse in the grounded circuit load.**

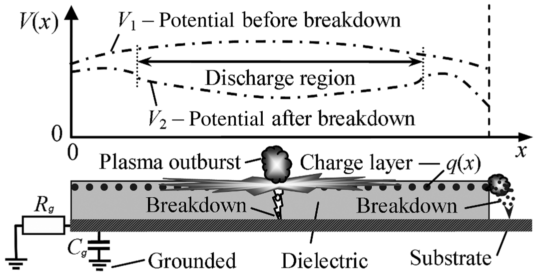


Fig. 4 **Types of discharges from dielectric (R–C denotes resistor–capacitor).**

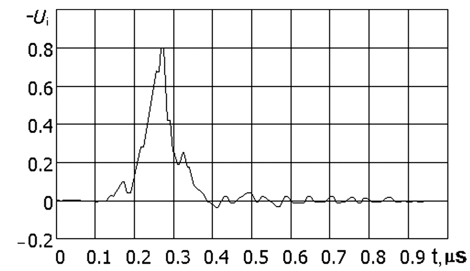


Fig. 5 **The typical form of the pulse during breakdown of a lavsan film (ρ = 60 μm).**

Figure 5 presents a pulse in the grounded circuit load during the lavsan film breakdown to the substrate. On the graph, the influence of discharges from other elements is seen, which is a system property for any group of discharges.

A multiple pulse can be initiated by the response pulse of structure impedance, external electromagnetic pulse, or induction between cables, as is shown in Fig. 6.

A system of discharges creates, in the structure of the SC model, a complicated response signal, as in Fig. 7.

The topology of solar panels creates a priori conditions for origin and propagation of discharges on sharp edges and fastening elements of plates. In the course of the tests, the discharges on a solar panel sample arose each 3–5 min. as splashes of white cathode spots with a frame of luminescence (Fig. 8).

The dielectric coatings of the radiators, as well as the thin films on the substrates, are mediums for the development of discharges similar to a tree. The discharges on a lavsan coating (ρ = 60 μm) of metal foil are shown in Fig. 9.

In the course of the tests, the appearance of the discharges changed considerably. At first, they developed with branches at the edges to the substrate (Fig. 9a). With time, the charged plasma outbursts in the vacuum appeared in a central area (Fig. 9b). Prolonged tests gave burnthrough channels down to the substrate due to stationary arcs on the film (Fig. 9c). The development of discharges on the surface of

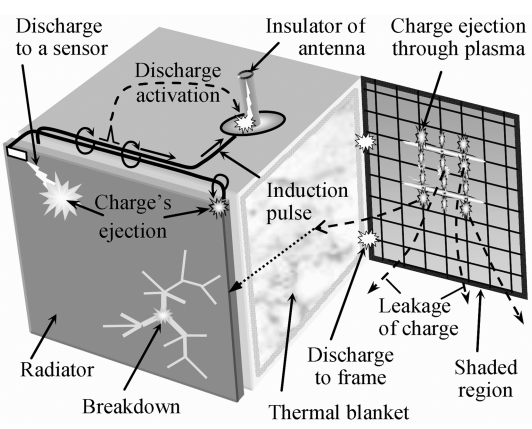


Fig. 6 **System of discharges on a model object.**

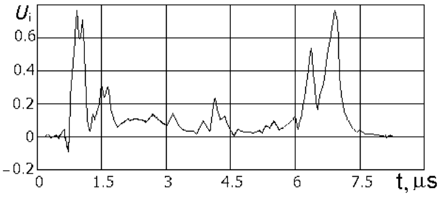
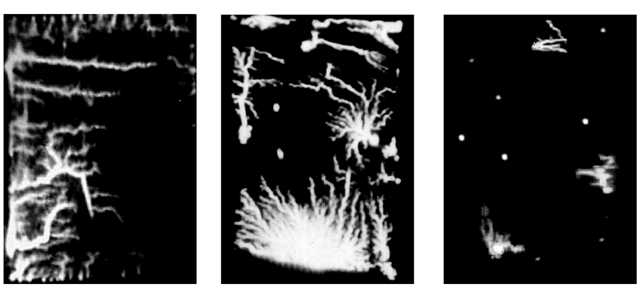


Fig. 7 System of discharges on a group of elements.

Zoom ×1 Zoom ×2 Zoom ×4

Fig. 8 Group discharges on a structure topology: $i_e \sim 10^{-9}$ A/cm^2 and $E_e = 20$ keV.



a) b) c)

Fig. 9 Change of discharge forms in the course of tests.

the sample is reflected by the summary signal packet of the response pulses in the grounded circuit (Fig. 10). Such signals, formed by paths along the edges and by branches on the surface (Fig. 9a), have a dynamic chaos type of structure and cannot be interpreted by only methods of spectral and statistic analyses. New information is required that can be obtained from the observation of the discharges and, particularly, from the identification of their topologic properties and organization of structure order.

Modeling of Discharges and Prospects for the General Theory

The analytical models of single discharges created on the basis of experiments [1–9] can be used for an analysis of linear forms of discharge development along the edges or at the interfaces between differently charged parts. But for the complicated discharges, the model of bifurcations should be coordinated by some external conditions: for example, by stochastic and heuristic decisions [10]. This simplification leads to distinction of the model ornament (Fig. 11a) from the self-coordinated structure of real discharge (Fig. 11b).

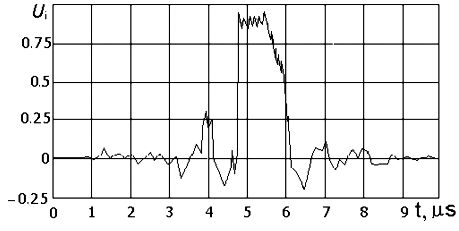
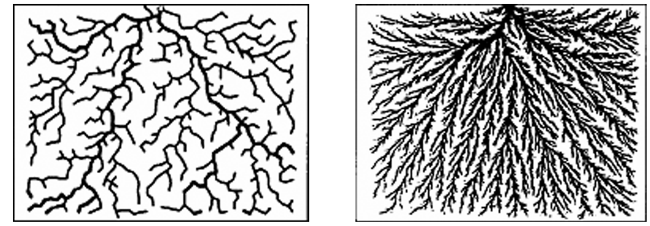


Fig. 10 A multiple discharge on the lavsan film.



a) b)

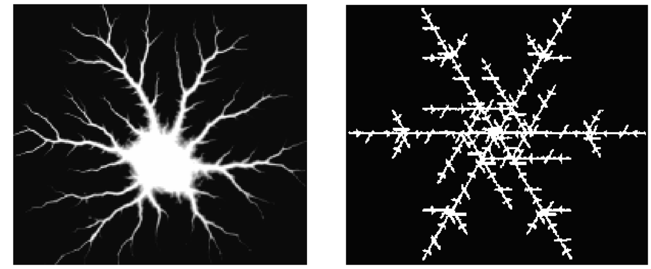
Fig. 11 Development of a) modeled and b) real processes.

The group isomorphism of discharges with the general equivalent single-type signal in its pulse group makes it possible to represent discharges as a chain of tensor transformations. The binding of the discharge group coordinates to the topology of their development and the peaks of the pulses (to shifts in time) allow decomposing of the complicated signal pulse into a series of peaks [4]. Having determined standard parameters for a group of pulses, we can synthesize the signal of complicated discharge by the scalar simplex of invariants in Eq. (1):

$$Y(t) = a_0 I_0 + a_1 I_1(t-\tau_1) + a_2 I_2(t-\tau_2) + \cdots \rightarrow Y(t) = \Sigma a_i \cdot I_i \rightarrow Y(t) = A \cdot J \quad (1)$$

The signal $Y(t)$ in formula (1) is the sum of discharge pulses in the grounded circuit and is also represented in the form of the product of level vector $A = (a_0, a_1, \ldots, a_n)$ by the vector of generalized pulse of standard forms $J = (I_0, I_1, \ldots, I_n)$. Both vectors are parameterized in the process by the vector of shift $S_\tau = [\tau_1, \tau_2, \ldots, \tau_i]$ for a countable series of events. An algorithm for the function $Y(t)$ in formula (1) can be simply found, but to obtain an analytical expression, complicated integral transformations of generalized functions are necessary. Only in rare instances, reduced to linear forms of operators, the function (1) can be written as a power or harmonic series.

The further evolution of expansion methods was wavelet images of pulse functions with the integral group of connection conditions. The progress in the description of pulse outbursts being beyond the reach of the function analysis gave a powerful impetus to the search for new methods of modeling processes. At the same time in topology, nonlinear objects with a fractal structure of treelike graphs, which relate as a graph item with Euler's lines, have appeared along with the classical objects of the linear homotopy groups. The problem of the conformity of the synthesis of the functions of an impulse for the topology of tracks is similar to a classical task of card colorations in different colors (known more than 150 years), but the general logical–algebraic solution for such a class of problems is still absent. The modeling task of complicated discharges sorts out the category of such problems, for which the logic of the interrelations of a track with a discharge has a system connection with their working out in parallel physical processes. Conditions of the beginning coordinates for each discharge are defined not only by the field in its nearest circle but by the influence of the fields of charges from the remote areas and the impulses of the discharges translating from the adjacent charge areas. The modern mathematics cannot solve such



a) Discharge **b) Dendrite**

Fig. 12 Topology of different processes in nature.

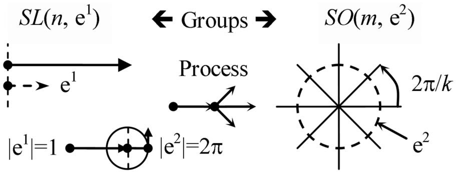


Fig. 13 Description of discharge track topology.

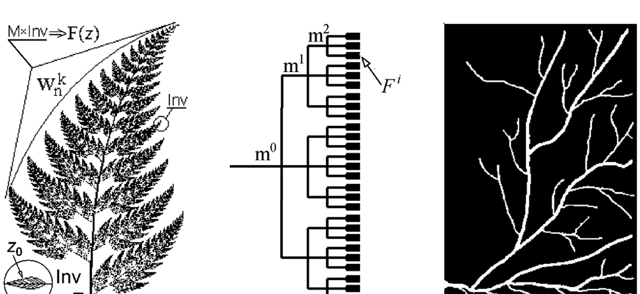
problems; the logic algorithms do not have any inverse solutions for the forecast, and their convergence in the iteration of approximations causes many difficulties. The stable connection between the topology and the order of processes in the group of isomorphism functions with invariants of an elementary physical process is the main property of development for nature phenomena that ensures the coordination of interconnections in similar structures: for example, in molecules. The structures in Fig. 12 show the similarity between the discharges and the growth of dendrite.

The general character of treelike topology for stable nature processes allows the taking of this form as an addition to the linear one. Therefore, the topology of discharge can be written as a complex group of elements of linear shift $SL(n, \mathrm{e}^1)$ and turn $SO(m, \mathrm{e}^2)$: that is, as the Diophantine equation $D(r) = n \cdot \mathrm{e}^1 + m \cdot \mathrm{e}^2$. The operations of *n* shifts and *m* phases of *k* bifurcations on the topology of the development of discharge tracks are shown in Fig. 13.

The use of a complex topology group extends the operation groups for the algebra of linear forms in Eq. (1) for discharge modeling by introducing the group of bifurcation into the model. Both groups of shift $SL(n, \mathrm{e}^1)$ and turns $SO(m, \mathrm{e}^2)$ are isomorphic to the countable series $N^0 = [1, 2, \ldots, n]$ that is the common ideal of their univocal interrelations set in the recursive function of a complex group of topology and order. To specify a function for the bifurcation process sequence, the space of countable number series N^0 must be extended by the logic of a transfinite procedure. That superspace $N^T = [N^0 \times N^1 \times \cdots \times N^N]$ is of potency $\aleph^{\aleph}$: that is, more than the potency of continuum c. The cortege $\{T, N, W\}$ of topology T of order N and algebra W of operations over object properties uniquely determines the recursive function with potency c that is equivalent to a series of tensor transformations. In the expanded space N^T, the metatensors of the functions generate a recurrent form of embedded relations. The deductive definition of hyperspace in combination with mathematical objects of various classes allows us to uniformly describe connections in the complicated processes to multiple discharge models between processes in a complicated discharge for development of discharge models in the form of recursive tensor functions.

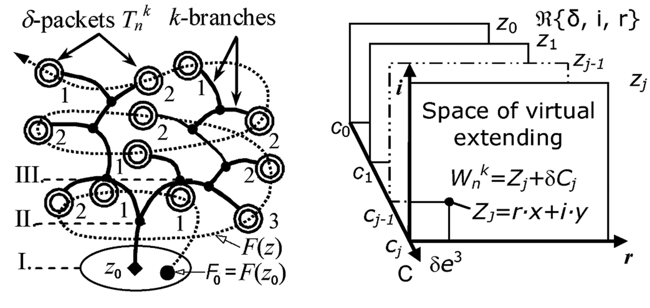
Space of Discharge Modeling

Riemann [11] has defined a generalized notion of space as an ordered system of points with similar properties that can be used for characterizing objects and their relations. To build a hyperspace, we should find a natural example that has stable properties similar to those of the model object. The model of a fern leaf (Fig. 14a) has its existential quantifier de facto (the problem for the abstract algebra), and its structure is stable in the course of hundreds millions of years (the empirical Poincaré condition is fulfilled). The fractal structure of the leaf (Fig. 14a) has a scale function $M = \mu^k$, where μ is the relation between properties of the element Inv (topological invariant of structure) and of the object on the nesting level k (as a matryoshka). In the topology of the leaf, the gluing of many branches creates (on the final level of nesting) a base field of complex plane Z_0 of element Inv for the embedded recursive biofunctions $F(z)$ on the Riemann surface. A similar structure has the configuration of a database (Fig. 14b) with the function of information storage $F(z)$. The community of the leaf topology (Fig. 14a), the database (Fig. 14b), and the discharge (Fig. 14c) allows us to replace function F transitive by the discharge pulse I in a local cell and to obtain a model of discharge. The philosophic-mathematical approach to the problem of the discharge description from the position of the Poincaré analysis situs has convinced the authors of the necessity of using subspaces of different classes: the structure of connections (topology), order relations (numbers), and the algebra of functions (extent of field). In the Poincaré prognosis, this space is defined as a general system of mathematics. But for that, it is necessary, as he had written, "to move away from the lantern of habitual views and to seek where nobody wants" [12].



a) Leaf **b) Structure** **c) Discharge**

Fig. 14 Topology examples of embedded objects.



a) **b)**

Fig. 15 The scheme of hyperspace representation.

The general mathematics uses operations with models of cognitive properties in various systems of nature (see Figs. 12 and 14). The hyperspace in Fig. 15 determines the levels of nesting $n \in N^0$ on the tree topology (Fig. 15a) and the addresses of the terminals T_n^k on k branches that carry the space $\Re\{\delta, i, r\}$ (Fig. 15b) with the function W_n^k (Fig. 15).

The transfinite hierarchy of the field of integer numbers in the class N^T of potency $\aleph^{\aleph}$ indicates the addresses of the terminals in the VCS. On the field $\Re$ (Fig. 15b), the leaves of δ packets are equivalent to the extension of the analytical function as a Riemann surface on the complex plane Z_0. The shift group $SL(j, \mathrm{e}^3)$ {introduced by W. Clifford on the basis of virtual numbers ($\delta = \sqrt{}0$) [13]} is isomorphic to the circumferential group $SO(j, \mathrm{e}^2)$ on the leaves of δ packets $W_n^k \Rightarrow \Re$, which enriches the operation variety of algebra. The complicated connections of function $F(z)$ in VCS (see Fig. 15a) determine, on the $\Re$ field, operation indexes of addresses for tensor groups with invariants of basis equivalents. Therefore, the successions of metatensors are reflexive with the similarity function by the group of connections, and there exists a minimum series equivalent to the whole group. The chains of semigroups of logical–algebraic operators on the topology of VCS addresses correspond with the group of homotopy on the sphere. Thus, if the chain of operators in Fig. 15a is moved from one branch to another (as the garland on a fir tree), the function of the chain remains whole. The signature of standard discharges can be determined by physical equivalents of structure samples with identification parameters during tests in experimental installations. A comparison of the results of various experiments with standard samples of discharge allows us to disclose their root distinctions and to include them in the general standards of the data to create the homogeneous knowledge base in the expert system for discharge identification [14].

Conclusions

Electrostatic charging modeling for SC materials and structure elements in ground vacuum installations shows inconsistency in the traditional mathematics potency for the description of ESDs. In accordance with the forecast of Poincaré about the development of general mathematics and coordination of its basic divisions (topology, organization, and functions), a method of ESD description in the cortège form of generalized parameters (topology of streamers, order of sequence, and current pulse functions) is proposed. In the expanded field of operations, the coordination of the identification of cortège parameters is achieved by the natural property of enclosed topomorphic structure conformity with isomorphic functions of processes on the branches of named topology structures.

In such a space, the description of logic and the topology of fractals are in cross correlation with the function of physical properties of their elements on the expanded Riemann's surface. The potency of this space is greater than that of continuum and is equivalent to the space of thinking; therefore, all objects in it are described unambiguously and can be completely identified by stable generalized features.

This approach ensures the mathematical modeling of discharge pulses and the creation of efficient methods of their identification for various experimental conditions.

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